

EXERCISE 5.3

In Problems 1–8, find exact solutions over the indicated intervals (x real and θ in degrees).

1. $2 \cos x + 1 = 0$, $0 \leq x < 2\pi$
2. $2 \sin x + 1 = 0$, $0 \leq x < 2\pi$
3. $2 \cos x + 1 = 0$, all real x
4. $2 \sin x + 1 = 0$, all real x
5. $\sqrt{2} \sin \theta - 1 = 0$, $0^\circ \leq \theta < 360^\circ$
6. $2 \cos \theta - \sqrt{3} = 0$, $0^\circ \leq \theta < 360^\circ$
7. $\sqrt{2} \sin \theta - 1 = 0$, all θ
8. $2 \cos \theta - \sqrt{3} = 0$, all θ

In Problems 9–14, solve each to four decimal places (x real and θ in degrees).

9. $4 \tan \theta + 15 = 0$, $0^\circ \leq \theta < 180^\circ$
10. $2 \tan \theta - 7 = 0$, $0^\circ \leq \theta < 180^\circ$

11. $5 \cos x - 2 = 0$, $0 \leq x < 2\pi$
12. $7 \cos x - 3 = 0$, $0 \leq x < 2\pi$
13. $5.0118 \sin x - 3.1105 = 0$, all real x
14. $1.3224 \sin x + 0.4732 = 0$, all real x

B For Problems 15–26, find exact solutions (x real and θ in degrees).

15. $\cos x = \cot x$, $0 \leq x < 2\pi$
16. $\tan x = -2 \sin x$, $0 \leq x < 2\pi$
17. $\cos^2 \theta = \frac{1}{2} \sin 2\theta$, all θ
18. $2 \sin^2 \theta + \sin 2\theta = 0$, all θ
19. $\tan(x/2) - 1 = 0$, $0 \leq x < 2\pi$
20. $\sec(x/2) + 2 = 0$, $0 \leq x < 2\pi$
21. $\sin^2 \theta + 2 \cos \theta = -2$, $0^\circ \leq \theta < 360^\circ$
22. $2 \cos^2 \theta + 3 \sin \theta = 0$, $0^\circ \leq \theta < 360^\circ$

23. $\cos 2\theta + \sin^2 \theta = 0$, $0^\circ \leq \theta < 360^\circ$
24. $\cos 2\theta + \cos \theta = 0$, $0^\circ \leq \theta < 360^\circ$
25. $4 \cos^2 2x - 4 \cos 2x + 1 = 0$, $0 \leq x \leq 2\pi$
26. $2 \sin^2(x/2) - 3 \sin(x/2) + 1 = 0$, $0 \leq x \leq 2\pi$

Solve Problems 27–30 (x real and θ in degrees). Compute inverse functions to four significant digits.

27. $4 \cos^2 \theta = 7 \cos \theta + 2$, $0^\circ \leq \theta \leq 180^\circ$
28. $6 \sin^2 \theta + 5 \sin \theta = 6$, $0^\circ \leq \theta \leq 90^\circ$
29. $\cos 2x + 10 \cos x = 5$, $0 \leq x < 2\pi$
30. $2 \sin x = \cos 2x$, $0 \leq x < 2\pi$

Solve Problems 31 and 32 for all real solutions. Compute inverse functions to four significant digits.

$$31. \cos^2 x = 3 - 5 \cos x \quad 32. 2 \sin^2 x = 1 - 2 \sin x$$

33. Explain the difference between evaluating the expression $\cos^{-1}(-0.7334)$ and solving the equation $\cos x = -0.7334$.

34. Explain the difference between evaluating the expression $\tan^{-1}(-5.377)$ and solving the equation $\tan x = -5.377$.

C Find exact solutions to Problems 35–38. [Hint: Square both sides at an appropriate point, solve, then eliminate any extraneous solutions at the end.]

35. $\sin x + \cos x = 1$, $0 \leq x < 2\pi$
36. $\cos x - \sin x = 1$, $0 \leq x < 2\pi$
37. $\sec x + \tan x = 1$, $0 \leq x < 2\pi$
38. $\tan x - \sec x = 1$, $0 \leq x < 2\pi$

Exercise 5.3

1. $2\pi/3, 4\pi/3$
3. $2\pi/3 + 2k\pi, 4\pi/3 + 2k\pi, k$ any integer
5. $45^\circ, 135^\circ$
7. $45^\circ + k(360^\circ), 135^\circ + k(360^\circ), k$ any integer
9. 104.9314° 11. $1.1593, 5.1239$
13. $0.6696 + 2k\pi, 2.4720 + 2k\pi, k$ any integer
15. $\pi/2, 3\pi/2$
17. $90^\circ + k(180^\circ), 45^\circ + k(180^\circ), k$ any integer
19. $\pi/2$ 21. 180° 23. $90^\circ, 270^\circ$
25. $\pi/6, 5\pi/6, 7\pi/6, 11\pi/6$ 27. 104.5°
29. $0.9987, 5.284$
31. $0.9987 + 2k\pi, -0.9987 + 2k\pi, k$ any integer
33. $\cos^{-1}(-0.7334)$ has exactly one value, 2.3941 ; the equation $\cos x = -0.7334$ has infinitely many solutions, which are found by adding $2\pi k, k$ any integer, to each solution in one period of $\cos x$.
35. $0, \pi/2$ 37. 0 39. 0.002613 sec 41. 33.21°
43. 64.1° 45. $(r, \theta) = (1, 30^\circ), (1, 150^\circ)$
47. $\theta = 45^\circ$